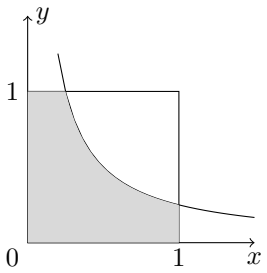
has real solutions?

Solution:

The equation has real roots if and only if:

$$1 - 4AB > 0 \quad \text{i.e.} \quad AB < \frac{1}{4}.$$

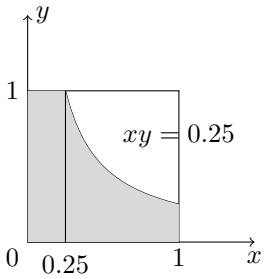
This area is shown here:



Since (A, B) is uniformly chosen in the square we can say that probability of having real roots is

$$\begin{aligned} P(R) &= \frac{\text{area of the shaded region}}{\text{area of the square}} \\ &= \frac{\text{area of the shaded region}}{1} \end{aligned}$$

To find the area of the shaded region we can set up the following integral:



$$\begin{aligned} Area &= \frac{1}{4} + \int_{\frac{1}{4}}^{1} \frac{1}{4x} dx \\ &= \frac{1}{4} + \frac{1}{4} \left[\ln(x)\right]_{\frac{1}{4}}^{1} \\ &= \frac{1}{4} + \frac{1}{4} \ln 4 \end{aligned}$$