## Item 1

Lad $A$ og $B$ være følgende matricer:

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 3 & 1 & 0 \\ 2 & 3 & -3 \end{bmatrix}, \text{ og } B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 3 & -1 \end{bmatrix}$$

a. Løs ligningen $AX = B$. Angiv dine input som fire heltal mellem 0 og 99.

---

Let A and B be the following matrices:

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 3 & 1 & 0 \\ 2 & 3 & -3 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 3 & -1 \end{bmatrix}$$

a. Solve the equation $AX = B$. State your inputs as four integers between 0 and 99.

1 $$\begin{bmatrix} \square & \square \\ \square & -3 \\ \square & -2 \end{bmatrix}$$

Correct answers:

1 $$\begin{bmatrix} 0 & 1 \\ 1 & -3 \\ 0 & -2 \end{bmatrix}$$

b. Overvej nu denne lille ændring til $A$

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 3 & k & 0 \\ 2 & 3 & -3 \end{bmatrix}$$

Find værdien af $k$, så ligningen $AX = B$ ikke har nogen løsning. Angiv dine input som to heltal mellem 0 og 99, så svaret er en irreducerbar brøk.

b. Now consider this small modification to A

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 3 & k & 0 \\ 2 & 3 & -3 \end{bmatrix}$$

Find the value of k such that the equation $AX = B$ has no solution. State your inputs as two integers between 0 and 99 such that the answer is an irreducible fraction.

1 $\frac{\square}{\square}$

Correct answers:

1 $\frac{18}{5}$

## Item 2

Givet det følgende lineære ligningssystem:

$$\begin{cases} x_1 + 8x_3 + 6x_4 = 0 \\ 2x_1 + 3x_2 - x_3 + 4x_4 = 0 \\ 4x_1 + 5x_2 + 7x_4 = 0 \end{cases}$$

a. Skriv totalmatricen. Angiv input som ni heltal mellem 0 og 99.

Given the following system of linear equations:

$$\begin{cases} x_1 + 8x_3 + 6x_4 = 0 \\ 2x_1 + 3x_2 - x_3 + 4x_4 = 0 \\ 4x_1 + 5x_2 + 7x_4 = 0 \end{cases}$$

a. Write the augmented matrix. Give the inputs as nine integers between 0 and 99.

1 $$\begin{bmatrix} 1 & \square & \square & 6 & \square \\ 2 & \square & -1 & \square & 0 \\ \square & 5 & \square & \square & \square \end{bmatrix}$$

Correct answers:

1 $$\begin{bmatrix} 1 & 0 & 8 & 6 & 0 \\ 2 & 3 & -1 & 4 & 0 \\ 4 & 5 & 0 & 7 & 0 \end{bmatrix}$$

b. Løs systemet og skriv løsningen i parametrisk vektorform. Angiv dine input som fire heltal mellem 0 og 99.

b. Solve the system and write the solution in parametric vector form. State your inputs as four integers between 0 and 99.

1 $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_{\square} \begin{bmatrix} \square \\ -\square \\ -1 \\ \square \end{bmatrix}$$

Correct answers:

1 $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_4 \begin{bmatrix} 2 \\ -3 \\ -1 \\ 1 \end{bmatrix}$$

## Item 3

Angiv, om følgende matricer er i reduceret række-echelonform (RREF) eller ej. Bemærk, at du i denne opgave skal have mere end halvdelen korrekt for at opnå point.

Mark whether the following matrices are in reduced row echelon form (RREF) or not. Note, in this assignment you must have more than half correct in order to obtain points.

| | | RREF | **Not** RREF |
|---|---|---|---|
| A | $\begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ | ○ ✓ | ○ |
| B | $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | ○ ✓ | ○ |
| C | $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ | ○ | ○ ✓ |
| D | $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ | ○ | ○ ✓ |
| E | $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | ○ ✓ | ○ |
| F | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | ○ ✓ | ○ |
| G | $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$ | ○ ✓ | ○ |
| H | $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | ○ | ○ ✓ |

## Item 4

Hvor mange korthænder på 4 kort, hvor rækkefølgen ikke betyder noget, kan gives fra et standardspil, så hånden indeholder mindst én spar?

How many card hands of 4 cards, where order does not matter, can be dealt from a standard deck such that the hand contains at least one spade?

1

Correct answers:

1 $188474$

## Item 5

80% af danskerne går regelmæssigt til lægen; 35% af dem har ingen helbredsproblemer i det følgende år. Af de resterende danskere har kun 5% ingen helbredsproblemer i det følgende år.

Hvad er sandsynligheden for, at en tilfældigt valgt dansker vil have helbredsproblemer i det følgende år?

Angiv dit svar som et heltal mellem 0 og 99, hvor du giver to decimalers præcision, korrekt afrundet.

80% of Danes go to the doctor regularly; 35% of those have no health issues during the following year. Of the remaining Danes, only 5% have no health issues during the following year.

What is the probability a randomly chosen Dane will have health issues in the following year?

State your answer as an integer between 0 and 99 such that you supply two decimal precision, correctly rounded off.

1

Correct answers:

1 $0.71$

## Item 6

Lad $A$ være mængden af 7-bit binære tal, og lad $B$ være mængden af binære tal, der slutter med 11.

a. Hvad er antallet af elementer i mængden $A \cap B$?

Skriv dit svar som et positivt heltal.

Let $A$ be the set of 7-bit binary numbers and let $B$ be the set of binary numbers ending with 11.

a. What is the number of elements in the set $A \cap B$?

Write your answer as a positive integer.

1

Correct answers:

1 32

b. Lad $x$ være et tal i mængden $A \cap B$. Hvad er sandsynligheden for, at $x = 23$ eller $x = 27$? Angiv dit svar som en irreducerbar brøk.

b. Let $x$ be a number in the set $A \cap B$. What is the probability that $x = 23$ or $x = 27$? State your answer as an irreducible fraction.

1

Correct answers:

1 $\frac{1}{16}$

## Item 7

300 fisk undersøges for DNA-defekter. Tabellen nedenfor viser resultaterne:

Environmental toxin found

DNA defects found

| | Yes | No | Total |
|---|---|---|---|
| Yes | 156 | 7 | 163 |
| No | 5 | 132 | 137 |
| Total | 161 | 139 | 300 |

Hvad er sandsynligheden for, at en fisk har en defekt, givet at miljøgifte blev fundet?

Angiv dit svar som et heltal mellem 0 og 99, hvor du giver to decimalers præcision, korrekt afrundet.

300 fish are examined for DNA defects. The table below shows the result:

Environmental toxin found

DNA defects found

| | Yes | No | Total |
|---|---|---|---|
| Yes | 156 | 7 | 163 |
| No | 5 | 132 | 137 |
| Total | 161 | 139 | 300 |

What is the probability that a fish has a defect given that environmental toxin was found?

State your answer as an integer between 0 and 99 such that you supply two decimal precision, correctly rounded off.

1

Correct answers:

1 0.97

Item 8

```
Algorithm loop1(n)
  s = 1
  for i = n to 1 step −1
      s = s + 1

Algorithm loop2(n)
  for i = 1 to n
      j = i
      while j > 0
          j = j − 1

Algorithm loop3(n)
  s = 0
  i = n
  while i > 0
      for j = 1 to i
          s = s + 1
      i = i − 1

Algorithm loop4(n)
  i = 0
  j = 0
  while i ≤ n
      if i < j then
          i = i + 1
      else
          j = j + 1
          i = 0
```

For hver af de ovenstående algoritmer, angiv deres køretid som en funktion af $n$ i $\Theta$-notation.


```
Algorithm loop1(n)
  s = 1
  for i = n to 1 step −1
      s = s + 1

Algorithm loop2(n)
  for i = 1 to n
      j = i
      while j > 0
          j = j − 1

Algorithm loop3(n)
  s = 0
  i = n
  while i > 0
      for j = 1 to i
          s = s + 1
      i = i − 1

Algorithm loop4(n)
  i = 0
  j = 0
  while i ≤ n
      if i < j then
          i = i + 1
      else
          j = j + 1
          i = 0
```


For each of the above algorithms, state its execution time as a function of $n$ in $\Theta$-notation.

| | $\Theta\left(n^3\right)$ | $\Theta\left((\log n)^2\right)$ | $\Theta(\sqrt{n})$ | $\Theta\left(n^2\right)$ | $\Theta(n \log n)$ | $\Theta(n\sqrt{n})$ |
|---|---|---|---|---|---|---|
| loop1 | ○ | ○ | ○ | ○ | ○ | ○ |
| loop2 | ○ | ○ | ○ | ○ ✔ | ○ | ○ |
| loop3 | ○ | ○ | ○ | ○ ✔ | ○ | ○ |
| loop4 | ○ | ○ | ○ | ○ ✔ | ○ | ○ |

## Item 9

Overvej ordet VIRGINIA. Hvor mange unikke måder kan disse bogstaver arrangeres på? Angiv dit svar som et positivt heltal.

Consider the word, VIRGINIA. How many unique ways can these letters be arranged? State your answer as a positive integer.

1

Correct answers:

1 $6720$

## Item 10

Et IT-firma modtager sine trykte kredsløbskort fra to forskellige leverandører, 1 og 2. Registreringer viser, at 5% af kredsløbskortene fra leverandør 1 og 3% af kredsløbskortene fra leverandør 2 er defekte. 60% af firmaets nuværende kredsløbskort kommer fra leverandør 2, og resten fra leverandør 1. Firmaet holder normalt et lager på 2000 kredsløbskort.

a. Baseret på disse oplysninger, konstruer en kontingenstabel over virksomhedens lager af kredsløbskort. Indsæt værdierne nedenfor.

---

An IT company receives its printed circuit boards from two different suppliers, 1 and 2. Records show that 5% of the circuit boards from supplier 1 and 3% of the circuit boards from supplier 2 are defective. 60% of the company's current circuit boards come from supplier 2, and the remaining from supplier 1. The company usually keeps a stock of 2000 circuit boards.

a. Based on this information, construct a contingency table of the company's circuit board stock. Insert the values below.

| | **Supplier 1** | **Supplier 2** |
|---|---|---|
| **Defectives** | 1 | 2 |
| **Non-Defectives** | 3 | 4 |

Correct answers:

1 $40$ 2 $36$ 3 $760$ 4 $1164$

b. Hvis et tilfældigt valgt kredsløbskort fra virksomhedens lager viser sig at være defekt, hvad er sandsynligheden for, at kredsløbskortet kommer fra leverandør 1? Angiv dit svar som en sandsynlighed med 4 decimalers precision, fx 0.1234. Husk at bruge punktum som decimal seperator: "."

---

b. If a randomly chosen circuit board from the company's stock is chosen and turns out to be defective, what is the probability that the circuit board is from supplier 1. State your answer with 4 decimal precision, e.g. 0.1234. Remember to use dot as decimal seperator: "."

1

Correct answers:

1 $0.5263157894736842105$

## Item 11

Sygdom *A* forekommer med sandsynlighed 0.1, og sygdom *B* forekommer med sandsynlighed 0.2. Det er ikke muligt at have begge sygdomme. Du har én test. Denne test rapporterer positiv med sandsynlighed 0.8 for en patient med sygdom *A*, med sandsynlighed 0.5 for en patient med sygdom *B*, og med sandsynlighed 0.01 for en patient uden sygdom - kald denne hændelse *W*. En positiv test angives som *P*.

Hvis testen viser sig positiv, hvad er sandsynligheden for, at du enten:

1. har sygdom *A*
2. har sygdom *B*, eller
3. har ingen af delene

Angiv dit svar som en sandsynlighed med 4 decimalers precision, fx 0.1234. Husk at bruge punktum som decimal seperator: "."

---

Disease *A* occurs with probability 0.1, and disease *B* occurs with probability 0.2. It is not possible to have both diseases. You have a single test. This test reports positive with probability 0.8 for a patient with disease *A*, with probability 0.5 for a patient with disease *B*, and with probability 0.01 for a patient with no disease - call the latter event *W*.

If the test comes back positive, what is the probability you have either:

1. disease *A*
2. disease *B*, or
3. neither

State your answer with 4 decimal precision, e.g. 0.1234. Remember to use dot as decimal seperator: "."

$P(A \mid P) =$ 1 [ ]

$P(B \mid P) =$ 2 [ ]

$P(W \mid P) =$ 3 [ ]

Correct answers:

1 $0.4278074866$ 2 $0.5347593583$ 3 $0.0374331551$

Item 12

Bestem den homogene løsning af coefficient matricen $A$ ved at opstille løsningen i parametrisk form:.

$$A = \begin{bmatrix} -1 & 2 & 1 & 4 \\ 1 & 2 & 2 & 6 \end{bmatrix}$$

Indsæt dine svar som positive heltal. Alle fortegn er fortrykte. Bemærk også at du skal indsættes indeks for de frie variable.

Determine the homogenuous solution for the coefficient matrix $A$ by setting up the solution in parametric form:

$$A = \begin{bmatrix} -1 & 2 & 1 & 4 \\ 1 & 2 & 2 & 6 \end{bmatrix}$$

Insert your answers as positive integers. All signs have been pre-printed. Also, note that you must also insert the index of the free variables.

1 $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_{\square} \begin{bmatrix} -\frac{1}{\square} \\ -\frac{3}{\square} \\ 1 \\ \square \end{bmatrix} + x_{\square} \begin{bmatrix} -\square \\ -\frac{\square}{2} \\ \square \\ \square \end{bmatrix}$$

Correct answers:

1 $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{4} \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ -\frac{5}{2} \\ 0 \\ 1 \end{bmatrix}$$

Item 13

For at opnå point i denne opgave skal du have 3 eller flere spørgsmål korrekte. **Ingen dokumentation er nødvendig.**

In order to obtain points in this assignments, you will need more have 3 or more items correct. **No documentation is needed.**

| | | True | False |
|---|---|---|---|
| A | $k=3$ vil gøre søjlerne i følgende matrix lineært uafhængige: $\begin{bmatrix} 2 & -10 & 6 \\ 0 & 1 & -2 \\ 0 & 0 & 2k-6 \end{bmatrix}$<br><br>$k=3$ will make the columns of the following matrix linearly independent: $\begin{bmatrix} 2 & -10 & 6 \\ 0 & 1 & -2 \\ 0 & 0 & 2k-6 \end{bmatrix}$ | ○ | ○ ✔ |
| B | $A\mathbf{x}=\mathbf{b}$ dannet ud fra følgende matrix har en løsning for hver: $\mathbf{b}$: $\begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 5 & -7 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$<br><br>$A\mathbf{x}=\mathbf{b}$ formed from the following matrix has a solution for each $\mathbf{b}$: $\begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 5 & -7 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ | ○ | ○ ✔ |
| C | Hvis søjlerne i *A* består af følgende vektorer, så er *A* invertibel: $u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}, u_4 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}.$<br><br>If the columns of *A* are made up of the following vectors, then *A* is invertible: $u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \end{bmatrix}, u_4 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}.$ | ○ ✔ | ○ |
| D | Vektorerne fra spørgsmål (c) udgør en uafhængig mængde.<br><br>The vectors from question (c) form an independent set. | ○ ✔ | ○ |
| E | Let $u_1,\ u_2,\ u_3, \text{and}\ u_4$ be as in question (c), then $u_4$ lies in span $\{u_1, u_2,\ u_3\}$.<br><br>Lad $u_1,\ u_2,\ u_3, \text{og}\ u_4$ være som i spørgsmål (c), så ligger $u_4$ i span $\{u_1, u_2,\ u_3\}$. | ○ | ○ ✔ |