Item 1

Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 9 \\ -3 & k \end{bmatrix}$

a. What value(s) of k, if any, will make $AB = BA$? (Note a negative sign is pre-printed)

b. For what value(s) of k are the columns of the following matrix linearly dependent?

$$\begin{bmatrix} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & k \end{bmatrix}$$

c. Find the value(s) of k for which the matrix $\begin{bmatrix} k^2 & 2k \\ 8 & k \end{bmatrix}$ is singular (i.e. not invertible).

a. State your answer as a positive integer

- 1 [ ]

Correct answers:

1 2

b. State your answer as a positive integer.

1 [ ]

Correct answers:

1 3

c. State your answer as a positive integer.

$\pm$ 1 [ ] and 2 [ ]

Correct answers:

1 4 2 0

Item 2

a.

Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \text{ and } \bar{b} = \begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}$$

a. Find the inverse of A using elementary row operations and the identity matrix.

b. Use the inverse of A to solve $A\bar{x} = \bar{b}$

a. State your answer as positive integers.

$$A^{-1} = \begin{bmatrix} -\square & \square & \square \\ -\square & \square & \square \\ 1 & \square & -\square \end{bmatrix}$$

Correct answers:

$$A^{-1} = \begin{bmatrix} -3 & 1 & 5 \\ -2 & 1 & 3 \\ 1 & 0 & -1 \end{bmatrix}$$

b. State your answer as positive integers.

$$\bar{x} = \begin{bmatrix} 1 \\ \square \\ \square \end{bmatrix}$$

Correct answers:

$$\bar{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Item 3

Consider the following linear system

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 1 \\ -2x_1 + x_2 + x_3 &= -5 \\ 2x_1 - x_2 - 2x_3 &= a \end{aligned}$$

For which values of a is the system consistent?

Choose the correct answer below.

A For all real values of a ✔

B For all real values of a **except** 5

C Only a =5

D There are no values of a that make the system consistent

E Only a = 0 and 5

F Only a =5 and -5

G Only a =0 and 1

Item 4

Let A be the matrix $A = \begin{bmatrix} 4 & 8 & -2 \\ -6 & 2 & 10 \\ -2 & 6 & 6 \end{bmatrix}$, and let $\vec{b}$ be the vector $\vec{b} = \begin{bmatrix} 2 \\ 18 \\ 15 \end{bmatrix}$

a. Determine whether $\vec{b}$ is in the span of the columns of A by finding echelon form of the augmented matrix.

b. Let $\vec{v}_1, \vec{v}_2$ and $\vec{v}_3$ denote the columns of the matrix A. Is the set of vectors $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ linearly independent or linearly dependent? If it is linearly dependent, find a linear dependency relation.

a. State your answer as integers and then select the correct statement below.

Echelon Form:

$$\begin{bmatrix} 2 & \square & -\square & 1 \\ \square & \square & \square & 3 \\ \square & \square & \square & 1 \end{bmatrix}$$

Correct answers:

$$\begin{bmatrix} 2 & 4 & -1 & 1 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A The system is inconsistent which means that **b** is not in the span of the columns of *A*. ✔

B The system is consistent which means that **b** is not in the span of the columns of *A*.

C The system is inconsistent which means that **b** is in the span of the columns of *A*.

D The system is consistent which means that **b** is in the span of the columns of *A*.

b. Select the correct item from the drop-down menu and then you may have to state the below dependence relation using positive integers as inputs.

The columns of A are linearly [1: ___]

Correct answers:

1 dependent

The dependence relation:

$$\square v_1 - v_2 + \square v_3 = 0$$

Correct answers:

$$3v_1 - v_2 + 2v_3 = 0$$

Item 5

a. Let three matrices be given by:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Find the determinant of matrix X in the following matrix equation $XA = XB + C$. State your answer as an integer between 0 and 99.

In Python, you can find the determinant of a `Sympy` Matrix `A` using `A.Det()` or `Det(A)`

$$\det\ X = \square$$

Correct answers:

$$\det\ X = 1$$

Item 6

Let A be the following matrix:

$$\begin{bmatrix} 1 & 3 \\ -2 & -8 \end{bmatrix}$$

a. Find numbers p and q, such that $A^2 = pA + qI$, where I is the 2×2 identity matrix. Notice a negative sign is preprinted

$p = -$ 1 [ ]

$q =$ 2 [ ]

Correct answers:

1 7 2 2

b. Let $B = A - tI$, where t is a scalar. For which values of t is B not invertible?

State your answer as a positive integer.

$$t = \pm \frac{\sqrt{\square}}{2} - \frac{7}{2}$$

Correct answers:

$t = \pm \frac{\sqrt{57}}{2} - \frac{7}{2}$

Item 7