Sektion 1

Consider the following system of equations

$$\begin{aligned} 2x_2 + 3x_3 + 4x_4 &= 1 \\ x_1 - 3x_2 + 4x_3 + 5x_4 &= 2 \\ -3x_1 + 10x_2 - 6x_3 - 7x_4 &= -4 \end{aligned}$$

a. Set up the coefficient matrix A of the above system. State your answers as integers between 0 and 99.

$$A = \begin{bmatrix} \square & \square & \square & \square \\ \square & -\square & \square & \square \\ -\square & \square & -\square & -\square \end{bmatrix}$$

b. At each step, determine the elementary row operation that transforms the former matrix into the latter:

$$\begin{bmatrix} 0 & 2 & 3 & 4 & 1 \\ 1 & -3 & 4 & 5 & 2 \\ -3 & 10 & -6 & -7 & -4 \end{bmatrix} \overset{step\ 1}{\sim} \begin{bmatrix} 1 & -3 & 4 & 5 & 2 \\ 0 & 2 & 3 & 4 & 1 \\ -3 & 10 & -6 & -7 & -4 \end{bmatrix} \overset{step\ 2}{\sim} \begin{bmatrix} 1 & -3 & 4 & 5 & 2 \\ 0 & 2 & 3 & 4 & 1 \\ 0 & 1 & 6 & 8 & 2 \end{bmatrix} \overset{step\ 3}{\sim}$$

$$\begin{bmatrix} 1 & -3 & 4 & 5 & 2 \\ 0 & 1 & 6 & 8 & 2 \\ 0 & 2 & 3 & 4 & 1 \end{bmatrix} \overset{step\ 4}{\sim} \begin{bmatrix} 1 & -3 & 4 & 5 & 2 \\ 0 & 1 & 6 & 8 & 2 \\ 0 & 0 & -9 & -12 & -3 \end{bmatrix} \overset{step\ 5}{\sim} \begin{bmatrix} 1 & -3 & 4 & 5 & 2 \\ 0 & 1 & 6 & 8 & 2 \\ 0 & 0 & 1 & \frac{4}{3} & \frac{1}{3} \end{bmatrix}$$

Note: Indexing is done from 1, i.e. the first column is denoted 1, and the first row is denoted 1. State your answers as integers between 0 and 99.

$step\ 1: r_{\square} \leftrightarrow r_{\square}$

$step\ 2: r_{\square} \rightarrow r_{\square} + \square r_{\square}$

$step\ 3: r_{\square} \leftrightarrow r_{\square}$

$step\ 4: r_{\square} \rightarrow r_{\square} - \square r_{\square}$

$$step\ 5: r_{\square} \to -\frac{1}{\square} r_{\square}$$

The reduced row echelon form of the matrix from (b) is

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{3} & \frac{1}{3} \end{bmatrix}$$

c. Write down the general solution to the system. (State your answers as integers between 0 and 99)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{\square}{3} + \frac{1}{\square} x_{\square} \\ \square \\ \frac{1}{3} - \frac{4}{\square} x_{\square} \\ x_{\square} \end{bmatrix}$$

d. The general solution to the above system in parametric vector form is thus (State your answers as integers between 0 and 99)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{\square}{3} \\ \square \\ \frac{1}{3} \\ \square \end{bmatrix} + x_{\square} \begin{bmatrix} \frac{1}{\square} \\ \square \\ -\frac{\square}{3} \\ \square \end{bmatrix}$$

Sektion 2

You are given the following system of equations

$$\begin{cases} x_1 - 2x_2 - 9x_4 = 8 \\ -x_2 + 4x_3 + 3x_4 = -1 \\ -2x_1 + x_2 + x_3 + 5x_4 = -8 \end{cases}$$

(a) Write the system as a vector equation.

$$x_1 \begin{bmatrix} \square \\ \square \\ -\square \end{bmatrix} + x_2 \begin{bmatrix} -\square \\ -\square \\ \square \end{bmatrix} + x_3 \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} + x_4 \begin{bmatrix} -\square \\ \square \\ \square \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ -8 \end{bmatrix}$$

(b) Write the system as a matrix equation.

$$\begin{bmatrix} \square & -\square & \square & -\square \\ \square & -\square & \square & \square \\ -\square & \square & \square & \square \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \\ -8 \end{bmatrix}$$

The reduced row echelon form of the augmented matrix for the above system of equations is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 5 & -3 \\ 0 & 0 & 1 & 2 & -1 \end{bmatrix}$$

(c) Mark each of the following statements as true or false. Hint: Use the reduced row echelon form and the answers from (a) and (b).

The system is consistent	○ True	○ False
There is a unique solution to the system	○ True	○ False
$\text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -9 \\ 3 \\ 5 \end{bmatrix} \right\} = \mathbb{R}^3$	○ True	○ False
The set of vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -9 \\ 3 \\ 5 \end{bmatrix} \right\}$ is linearly independent	○ True	○ False
The solution set of the system of equations is a line in $\mathbb{R}^3$	○ True	○ False
$\begin{bmatrix} -9 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} + 5 \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$	○ True	○ False

Sektion 3

For each of the sets of vectors below determine whether the set is linearly dependent or linearly independent. Notice: you do not need to do a lot of calculations.

		Linearly dependent	Linearly independent
A	$\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix} \right\}$	○	○
B	$\left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -4 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$	○	○
C	$\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \end{bmatrix} \right\}$	○	○
D	$\left\{ \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 9 \\ 3 \end{bmatrix} \right\}$	○	○
E	$\left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$	○	○

(a) You are given the matrix $A = \begin{bmatrix} 3 & -2 & 0 \\ 1 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} -8 \\ 16 \\ 4 \end{bmatrix}$. Write $\mathbf{b}$ as a linear combination of the columns from A

$\mathbf{b} = -\square\mathbf{a}_1 + \square\mathbf{a}_2 + \square\mathbf{a}_3$

(b) You are given the matrix $A = \begin{bmatrix} 2 & -1 & 4 \\ 0 & 2 & 4 \\ 4 & -4 & 4 \end{bmatrix}$. Write $\mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}$ as a linear combination of the columns of A.

$\mathbf{b} = \square\mathbf{a}_1 + \square\mathbf{a}_2 - 2\mathbf{a}_3$

(c) You are given the matrix $A = \begin{bmatrix} 0 & 2 & 4 \\ 1 & -2 & -5 \\ 0 & 3 & 6 \end{bmatrix}$ and the vector $\mathbf{b} = \begin{bmatrix} h \\ 1 \\ 9 \end{bmatrix}$. Find the value of h such that $\mathbf{b}$ is a linear combination of the columns of A.

$h = \square$

Sektion 5

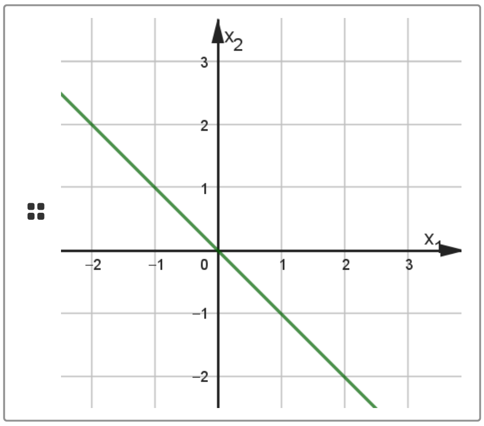
Below are given four augmented matrices in reduced row echelon form. Match each matrix with the geometric interpretation of the solution set.

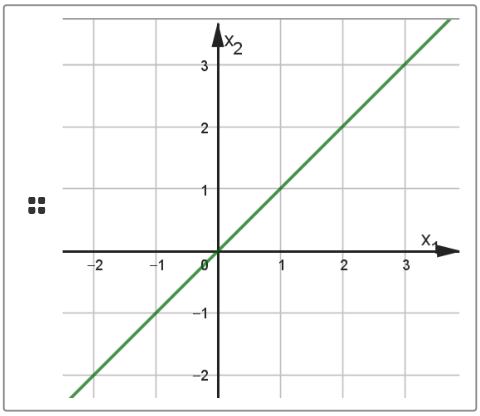
$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

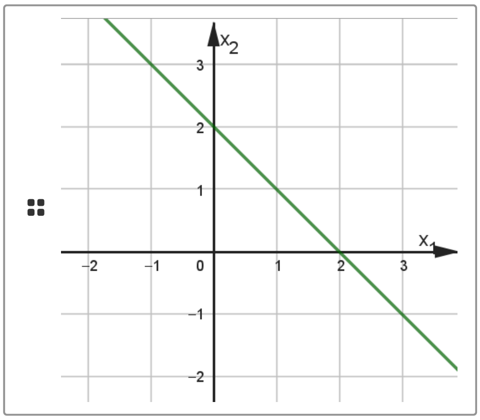
$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

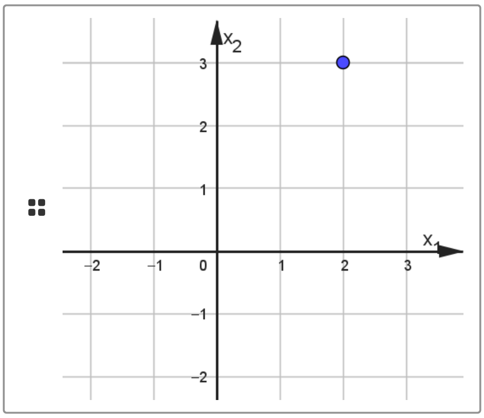
$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$









You are given a matrix $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ -2 & 0 & -2 & -2 \\ 1 & 2 & 3 & -1 \end{bmatrix}$ and a vector $\mathbf{b} = \begin{bmatrix} -2 \\ 4 \\ 8 \end{bmatrix}$.

(a) Write the general solution of $A\mathbf{x} = \mathbf{b}$ in parametric vector form. Note that you also need to fill in the subscripts of the free variables.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\square \\ \square \\ \square \\ \square \end{bmatrix} + x_{\square} \begin{bmatrix} -\square \\ -\square \\ 1 \\ \square \end{bmatrix} + x_{\square} \begin{bmatrix} -\square \\ \square \\ \square \\ 1 \end{bmatrix}$$

(b) Write the general solution to the homogenous equation $A\mathbf{x} = \mathbf{0}$ in parametric vector form

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_{\square} \begin{bmatrix} -\square \\ -\square \\ 1 \\ \square \end{bmatrix} + x_{\square} \begin{bmatrix} -\square \\ \square \\ \square \\ 1 \end{bmatrix}$$

(c) Mark each statement below as true or false. You might need to do some calculations.

$\begin{bmatrix} -2 \\ 5 \\ 0 \\ 0 \end{bmatrix}$ is a solution to the inhomogenous equation $A\mathbf{x} = \mathbf{b}$ from part (a)

○ True ○ False

$\begin{bmatrix} -2 \\ 5 \\ 0 \\ 0 \end{bmatrix}$ is a solution to the homogenous equation $A\mathbf{x} = \mathbf{0}$ from (b)

○ True ○ False

$\begin{bmatrix} 1 \\ 7 \\ -4 \\ 3 \end{bmatrix}$ is a solution to the homogenous equation $A\mathbf{x} = \mathbf{0}$ from (b)

○ True ○ False

$\begin{bmatrix} -1 \\ 12 \\ -4 \\ 3 \end{bmatrix}$ is a solution to the inhomogenous equation $A\mathbf{x} = \mathbf{b}$ from part (a)

○ True ○ False