Sektion 1

Information is transmitted through a fiber optic cable in the form of bits, which can have the value of 0 or 1. However, there is noise on the line, so a transmitted 0 will, with a probability of 1/10, be changed to 1 during transmission. A transmitted 1 will, with a probability of 1/5, be changed to 0 during transmission. The probability that a randomly chosen bit is transmitted with the value 0 is 2/3.

(a) A bit with the value 0 is received. What is the probability that the received bit was transmitted with the value 0?

1 [ ]

Correct answers:

1 0.9

(b) A bit with the value 1 is received. What is the probability that the received bit was transmitted with the value 1?

1 [ ]

Correct answers:

1 $\frac{4}{5}$

c) What is the probability that the recieved bit will be different from the transmitted bit?

1 [ ]

Correct answers:

1 $\frac{2}{15}$

Sektion 2

(a) State which form each of the below matrices are in.

	Row reduced echelon form	Row echelon form	No form
$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$	○	○ ✔	○
$\begin{bmatrix} 0 & -4 & 1 \\ 2 & 0 & 0 \\ 1 & -3 & 3 \end{bmatrix}$	○	○	○ ✔
$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 2 & -7 \end{bmatrix}$	○	○	○ ✔
$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$	○	○	○ ✔

(b) The following matrices are all augmented matrices in row echelon form. Find the free variables for each matrix.

i. $\begin{bmatrix} -3 & 0 & 2 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

- ☐ x_1
- ☐ x_2
- ☐ x_3 ✔
- ☐ There are no free variables

ii. $\begin{bmatrix} 1 & 2 & 0 & 0 & 9 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix}$

- ☐ x_1
- ☐ x_2 ✔
- ☐ x_3
- ☐ x_4 ✔
- ☐ There are no free variables

iii. $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

- [ ] x_1
- [ ] x_2
- [ ] x_3
- [ ] There are no free variables ✔

(c) Mark each of the following statements as true or false

The number of pivot columns in a reduced matrix is the same as the number of free variables.	○ TRUE	○ FALSE ✔
If a system is consistent, there will be at least one free variable	○ TRUE	○ FALSE ✔
The number of pivot columns determine the number of basic variables.	○ TRUE ✔	○ FALSE
If two matrices are row equivalent, they represent two systems of linear equations with the same set of solutions.	○ TRUE ✔	○ FALSE
It is possible for a system of linear equations to have exactly two solutions.	○ TRUE	○ FALSE ✔

Sektion 3

The following calculations should be done by hand.

Solve each of the following systems by writing down the augmented matrix and finding the reduced row echelon form.

(a) $\begin{cases} 5x_1 - 5x_2 + 5x_3 = 5 \\ 2x_1 + 4x_2 - 6x_3 = 12 \\ 10x_1 - 5x_2 + 5x_3 = 30 \end{cases}$

$x_1 =$ 1 ☐

$x_2 =$ 2 ☐

$x_3 =$ 3 ☐

Correct answers:

1 5 2 11 3 7

(b) $\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ 4x_1 + 5x_2 + 6x_3 = 0 \end{cases}$

$x_1 =$ 1 ☐ x_3

$x_2 =$ 2 ☐ x_3

x_3 free

Correct answers:

1 1 2 -2

Sektion 4

The following calculations should be done by hand.

(a) Given the system of linear equations

$$\begin{cases} x_1 + 2x_3 + 4x_4 = 6 \\ 4x_2 - 6x_3 - 3x_4 = 0 \\ 4x_1 + 8x_2 - 4x_3 + 10x_4 = 1 \end{cases}$$

Determine whether or not the system is consistent.

A The system is consistent since there is a pivot in each column of the reduced augmented matrix

B The system is consistent since there is a pivot in each column of the reduced coefficient matrix

C The system is inconsistent since there is a pivot in the rightmost column of the reduced augmented matrix ✔

D The system is inconsistent since there is a zero in the rightmost column of the reduced agumented matrix

(b) Given the system of equations

$$\begin{cases} x_1 - 2x_2 + 4x_3 = 1 \\ 2x_2 + x_3 = -5 \\ 2x_1 + 10x_3 = h \end{cases}$$

Find the value of h such that the system is consistent.

$h =$ 1 [ ]

Correct answers:

1 -8

Sektion 5

You may use Python to solve this assignment

Given three points $(1, 2)$, $(4, 5)$ and $(6, 4)$ find a second degree polynomial $p(t) = a_0 + a_1 t + a_2 t^2$ that passes through all three points. That is, find $a_0,\ a_1$ and a_2 to fulfill the following equations

$a_0 + a_1 \cdot 1 + a_2 \cdot 1^2 = 2$

$a_0 + a_1 \cdot 4 + a_2 \cdot 4^2 = 5$

$a_0 + a_1 \cdot 6 + a_2 \cdot 6^2 = 4$

$p(t) =$ 1 [ ] + 2 [ ] t + 3 [ ] t^2

Correct answers:

1 $-\frac{1}{5}$ 2 $\frac{5}{2}$ 3 $-\frac{3}{10}$

Sektion 6

Axel is twice as old as Bob. Caroline is 5 years older than Bob. The combined age of Axel, Caroline, and Danny is 110 years. Also, the sum of Axel and Bob's ages is the same as the sum of Caroline and Danny's ages.

(a) Which of the following system of equations correctly describe the situation

A

$$\begin{cases} A - 3B = 0 \\ B - C = 0 \\ A + C + D = 110 \\ A + B - C - D = 0 \end{cases}$$

B ✔

$$\begin{cases} A - 2B = 0 \\ B - C = -5 \\ A + C + D = 110 \\ A + B - C - D = 0 \end{cases}$$

C

$$\begin{cases} A + 2B = 0 \\ B - C = -5 \\ A + C + D = 110 \\ A + B + C + D = 0 \end{cases}$$

(b) Solve the system using python

Axel: 1 ☐ years old

Bob: 2 ☐ years old

Caroline: 3 ☐ years old

Danny: 4 ☐ years old

Correct answers:

1 44 2 22 3 27 4 39