Aflevering 6 Løsning

Opgave 1

A_0: Udsendt som 0

A_1: Udsendt som 1

B_0: Modtaget som 0

B_1: Modtaget som 1

Oplyst i opgaveteksten

$P(A_0) = \frac{2}{3}$

$P(A_1) = \frac{1}{3}$

$P(B_1 | A_0) = \frac{1}{10}$

$P(B_0 | A_0) = \frac{9}{10}$

$P(B_0 | A_1) = \frac{1}{5}$

$P(B_1 | A_1) = \frac{4}{5}$

(a) Find $P(A_0 | B_0)$

Bayes $P(A_0 | B_0) = \dfrac{P(B_0 | A_0) \cdot P(A_0)}{P(B_0)}$

Law of total probability

$$P(B_0) = P(B_0 | A_0) \cdot P(A_0) + P(B_0 | A_1) \cdot P(A_1)$$

$$= \frac{9}{10} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{1}{3}$$

$$= \frac{18}{30} + \frac{1}{15}$$

$$= \frac{9}{15} + \frac{1}{15} = \frac{10}{15} = \frac{2}{3}$$

$$P(A_0|B_0) = \frac{\frac{9}{10} \cdot \frac{2}{3}}{\frac{2}{3}} = \boxed{\frac{9}{10}}$$

(b) Find $P(A_1|B_1)$

Bayes $P(A_1|B_1) = \frac{P(B_1|A_1) \cdot P(A_1)}{P(B_1)}$

Law of total probability

$$P(B_1) = P(B_1|A_0) \cdot P(A_0) + P(B_1|A_1) \cdot P(A_1)$$
$$= \frac{1}{10} \cdot \frac{2}{3} + \frac{4}{5} \cdot \frac{1}{3}$$
$$= \frac{2}{30} + \frac{4}{15}$$
$$= \frac{1}{15} + \frac{4}{15}$$
$$= \frac{5}{15}$$
$$= \frac{1}{3}$$

$$P(A_1|B_1) = \frac{\frac{4}{5} \cdot \frac{1}{3}}{\frac{1}{3}} = \boxed{\frac{4}{5}}$$

(c) Find ssh for fejl

$$P(\text{Fejl}) = P(B_1|A_0) \cdot P(A_0) + P(B_0|A_1) \cdot P(A_1)$$
$$= \frac{1}{10} \cdot \frac{2}{3} + \frac{1}{5} \cdot \frac{1}{3}$$
$$= \frac{2}{30} + \frac{1}{15} = \frac{1}{15} + \frac{1}{15} = \boxed{\frac{2}{15}}$$

Opgave 2

(a) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\boxed{\text{Række echelon form}}$

$\begin{bmatrix} 0 & -4 & 1 \\ 2 & 0 & 0 \\ 1 & -3 & 3 \end{bmatrix}$ $\boxed{\text{Ingen form}}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 2 & -7 \end{bmatrix}$ ← Række af 0'er midt i matricen så $\boxed{\text{Ingen form}}$

$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ $\boxed{\text{Ingen form}}$

pivoter

(b) $\left[\begin{array}{ccc|c} -3 & 0 & 2 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\boxed{x_3}$ er en fri variabel

pivot pivot

$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 9 \\ 0 & 0 & 1 & 0 & 3 \end{array}\right]$

$\boxed{x_2}$ og $\boxed{x_4}$ er frie variable

$$\begin{array}{c} \begin{array}{ccc} \text{pivoter} & & \\ \downarrow & \downarrow & \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

$\boxed{\text{Ingen frie variable}}$

Opgave 3

(a)

$$5x_1 - 5x_2 + 5x_3 = 5$$
$$2x_1 + 4x_2 - 6x_3 = 12$$
$$10x_1 - 5x_2 + 5x_3 = 30$$

Totalmatrix

$$\begin{bmatrix} 5 & -5 & 5 & 5 \\ 2 & 4 & -6 & 12 \\ 10 & -5 & 5 & 30 \end{bmatrix} \begin{matrix} r_1 \to \frac{1}{5}r_1 \\ r_2 \to \frac{1}{2}r_2 \\ \sim \\ r_3 \to \frac{1}{5}r_3 \end{matrix} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 2 & -3 & 6 \\ 2 & -1 & 1 & 6 \end{bmatrix}$$

$$\underset{\sim}{r_2 \to r_2 - r_1} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 3 & -4 & 5 \\ 2 & -1 & 1 & 6 \end{bmatrix} \underset{\sim}{r_3 \to r_3 - 2r_1} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 3 & -4 & 5 \\ 0 & 1 & -1 & 4 \end{bmatrix}$$

$$\underset{\sim}{r_2 \leftrightarrow r_3} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -1 & 4 \\ 0 & 3 & -4 & 5 \end{bmatrix} \underset{\sim}{r_3 \to r_3 - 3r_2} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -1 & -7 \end{bmatrix}$$

$$\underset{\sim}{r_3 \to -1 \cdot r_3} \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & 7 \end{bmatrix} \begin{matrix} r_2 \to r_2 + r_3 \\ r_1 \to r_1 - r_3 \\ \sim \end{matrix} \begin{bmatrix} 1 & -1 & 0 & -6 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\underset{\sim}{r_1 \to r_1 + r_2} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\boxed{\begin{aligned} x_1 &= 5 \\ x_2 &= 11 \\ x_3 &= 7 \end{aligned}}$$

(b) $x_1 + 2x_2 + 3x_3 = 0$

$4x_1 + 5x_2 + 6x_3 = 0$

Total matrix

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \end{bmatrix} \overset{r_2 \to r_2 - 4r_1}{\sim} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix}$$

$$\overset{r_2 \to \frac{-1}{3} r_2}{\sim} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \overset{r_1 \to r_1 - 2r_2}{\sim} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

$x_1 - x_3 = 0 \iff x_1 = x_3$

$x_2 + 2x_3 = 0 \iff x_2 = -2x_3$

$x_3 = x_3$

Opgave 4

(a) Is the system consistent?

$$x_1 + 2x_3 + 4x_4 = 6$$

$$4x_2 - 6x_3 - 3x_4 = 0$$

$$4x_1 + 8x_2 - 4x_3 + 10x_4 = 1$$

Total matrix

$$\begin{bmatrix} 1 & 0 & 2 & 4 & 6 \\ 0 & 4 & -6 & -3 & 0 \\ 4 & 8 & -4 & 10 & 1 \end{bmatrix} \underset{r_3 \to r_3 - 4r_1}{\sim} \begin{bmatrix} 1 & 0 & 2 & 4 & 6 \\ 0 & 4 & -6 & -3 & 0 \\ 0 & 8 & -12 & -6 & -23 \end{bmatrix}$$

$$\underset{r_3 \to r_3 - 2r_2}{\sim} \begin{bmatrix} 1 & 0 & 2 & 4 & 6 \\ 0 & 4 & -6 & -3 & 0 \\ 0 & 0 & 0 & 0 & -23 \end{bmatrix}$$

$0 = -23$

så $\boxed{\text{ikke konsistent}}$

pga pivot i søjle længst mod højre i totalmatrix

(b) Find h så system er konsistent

$$x_1 - 2x_2 + 4x_3 = 1$$

$$2x_2 + x_3 = -5$$

$$2x_1 + 10x_3 = h$$

Totalmatrix

$$\begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 2 & 1 & -5 \\ 2 & 0 & 10 & h \end{bmatrix} \overset{r_3 \to r_3 - 2r_1}{\sim} \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 2 & 1 & -5 \\ 0 & 4 & 2 & h-2 \end{bmatrix}$$

$$\overset{r_3 \to r_3 - 2r_2}{\sim} \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 2 & 1 & -5 \\ 0 & 0 & 0 & h+8 \end{bmatrix}$$

Konsistent hvis $h+8=0 \iff \boxed{h=-8}$

Opgave 5

Punkter $(1,2)$, $(4,5)$, $(6,4)$

Model $p(t) = a_0 + a_1 t + a_2 t^2$

Ligninger

$a_0 + a_1 \cdot 1 + a_2 \cdot 1^2 = 2 \iff a_0 + a_1 + a_2 = 2$

$a_0 + a_1 \cdot 4 + a_2 \cdot 4^2 = 5 \iff a_0 + 4a_1 + 16a_2 = 5$

$a_0 + a_1 \cdot 6 + a_2 \cdot 6^2 = 4 \iff a_0 + 6a_1 + 36a_2 = 4$

Opgave 6

(a) Axel is twice as old as Bob

$$A = 2B \iff \boxed{A - 2B = 0}$$

Caroline is 5 years older than Bob

$$C = B + 5 \iff \boxed{B - C = -5}$$

The combined age of Axel, Caroline and Danny is 110 years.

$$\boxed{A + C + D = 110}$$

The sum of Axel and Bob's ages is the same as the sum of Caroline and Danny's ages

$$A + B = C + D \iff \boxed{A + B - C - D = 0}$$