Aflevering 5: Løsning

Opgave 1

A: mindst en 6'er i 6 terningekast

B: mindst to 6'ere i 12 terningekast

C: mindst tre 6'ere i 18 terningekast

$$P(A) = 1 - \left(\frac{5}{6}\right)^6 = \boxed{0.6651}$$

↑ ssh for ingen 6'ere i 6 kast

$$P(B) = 1 - \left(\frac{5}{6}\right)^{12} - \underbrace{12 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{11}} = \boxed{0.6187}$$

ssh for ingen 6'ere i 12 kast

ssh for præcis én 6'er i 12 kast

ssh for ingen 6'ere; ssh for én 6'er; ssh for to 6'ere

$$P(C) = 1 - \left(\frac{5}{6}\right)^{18} - 18 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{17} - C_2^{18} \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{16}$$

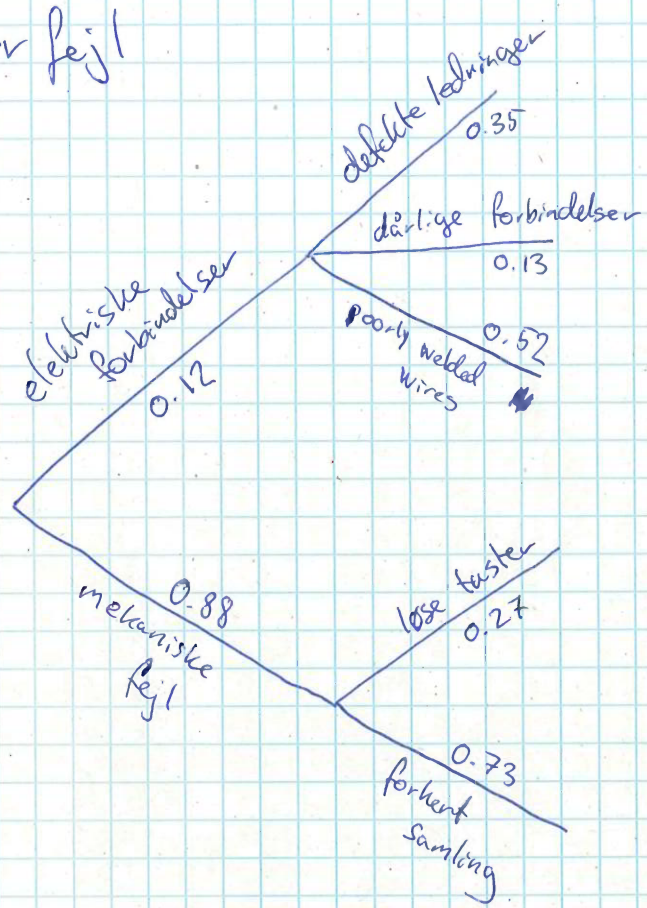
$$= 1 - \left(\frac{5}{6}\right)^{18} - 18 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{17} - \frac{18!}{2!\,16!} \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{16}$$

$$= \boxed{0.5973}$$

Blandt de tre hændelser A, B og C har hændelse **A størst sandsynlighed**

Opgave 2

Tastetur fejl



(a) ssh for at fejl skyldes løse taster

$$P(\text{løse taster}) = P(\text{løse taster} \mid \text{mek. fejl}) \cdot P(\text{mek. fejl})$$

$$= 0.27 \cdot 0.88$$

$$= \boxed{0.2376}$$

(b) ssh for at fejl skyldes dårlig forbindelse eller poorly welded wires

$$P(\text{dårlig forb} \cup \text{poorly welded}) = P(\text{dårlig forb}) + P(\text{poorly welded})$$

$$= P(\text{dårlig forb} \mid \text{el. forb.}) \cdot P(\text{el. forb}) + P(\text{poorly welded} \mid \text{el. forb}) \cdot P(\text{el. forb})$$

$$= 0.12 \cdot 0.13 + 0.12 \cdot 0.52$$

$$= \boxed{0.078}$$

Opgave 3

Antag $P(B|A) = 0.3$, $P(A) = 0.9$, $P(B|A^c) = 0.5$

(a) $P(B) = P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)$

law of total probability

$= 0.3 \cdot 0.9 + 0.5 \cdot 0.1$

$= \boxed{0.32}$

(b) $P(A|B) = \dfrac{P(B|A) \cdot P(A)}{P(B)}$

Bayes sætning

$= \dfrac{0.3 \cdot 0.9}{0.32}$

$= \boxed{0.8438}$

Opgave 4

Fly

D: afgår til tiden

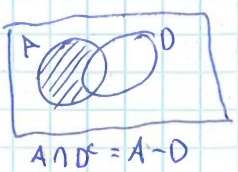
A: ankommer til tiden

$P(A) = 0.8$, $P(D) = 0.81$, $P(A \cap D) = 0.76$

$P(A|D^c) = \dfrac{P(A \cap D^c)}{P(D^c)} = \dfrac{P(A - D)}{P(D^c)} = \dfrac{P(A) - P(A \cap D)}{P(D^c)}$

$= \dfrac{0.8 - 0.76}{1 - 0.81}$

$= \boxed{0.21}$



Opgave 5

Email Spam

$P(\text{spam}) = 0.1$

$P(\text{"Dear Friend"} \mid \text{spam}) = 0.15$

$P(\text{"Dear Friend"} \mid \text{not spam}) = 0.01$

(a) $P(\text{"Dear Friend"}) = P(\text{"Dear Friend"} \mid \text{spam}) \cdot P(\text{spam}) + P(\text{"Dear Friend"} \mid \text{not spam}) P(\text{not spam})$

$= 0.15 \cdot 0.1 + 0.01 \cdot 0.9$

$= \boxed{0.024}$

(b) $P(\text{spam} \mid \text{"Dear Friend"}) = \dfrac{P(\text{"Dear Friend"} \mid \text{spam}) \cdot P(\text{spam})}{P(\text{"Dear Friend"})}$

$= \dfrac{0.15 \cdot 0.1}{0.024}$

$= \boxed{0.625}$

Opgave 6

(a)

	Rural	Suburban	Urban	Total
Grades	57	87	103	247
Learning	50	42	49	141
Social	42	22	26	90
Total	149	151	178	478

Beregninger:

Social Total: $478 - 247 - 141 = 90$

~~Learning Total:~~

Learning Urban: $141 - 50 - 42 = 49$

Social Urban: $178 - 103 - 49 = 26$

Total Suburban: $478 - 149 - 178 = 151$

Grades Rural: $\frac{247 - 103 - 30}{2} = \frac{114}{2} = 57$

Grades Suburban: $57 + 30 = 87$

Social Rural: $149 - 57 - 50 = 42$

Social Suburban: $90 - 42 - 26 = 22$

Tal: fra at løse

Tal: fra beregning

(b) $P(\text{Suburban} \cap \text{Social}) = \frac{22}{478} = \boxed{0.05}$

(c) $P(\text{Suburban} \mid \text{Social}) = \frac{22}{90} = \boxed{0.24}$

(d) $P(\text{Social} \mid \text{Suburban}) = \frac{22}{151} = \boxed{0.15}$

(e) Er 'Rural' og 'Grades' uafhængige?

$P(\text{Rural} \mid \text{Grades}) = \frac{57}{247} = 0.23$

$P(\text{Rural}) = \frac{149}{478} = 0.31$

Da $P(\text{Rural} \mid \text{Grades}) \neq P(\text{Rural})$ er hændelserne ikke uafhængige

Hvis dette gælder, gælder det også at $P(\text{Grades} \mid \text{Rural}) \neq P(\text{Grades})$ som er det du er en svar mulighed på WF