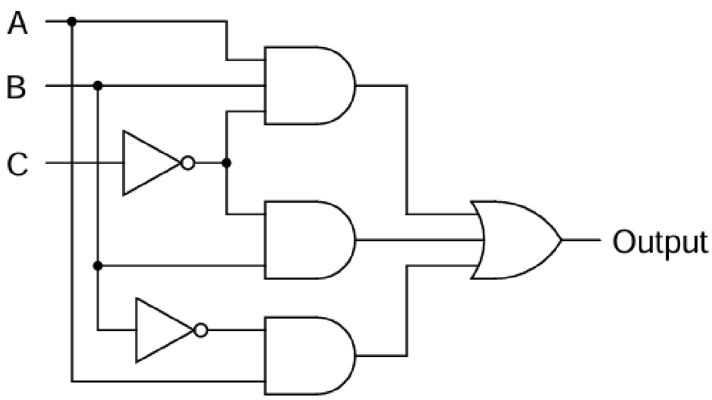
# MSE Hand in 4 - Solution

The assignments are to be solved in pairs. Each pair can only hand in one solution. Submission by the end of class. The assignments must be solved by hand.

## Assignment 1

These are recap exercises from last week's topic.

Consider the following logic gate circuit:



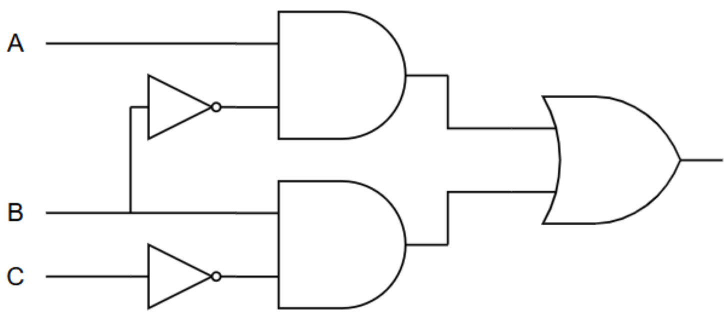

(a) State the Boolean function for the gate.

$F(A, B, C) = \underline{\underline{AB\bar{C} + B\bar{C} + \bar{B}A}}$

(b) Use Boolean algebra to simplify the logic gate circuit.

$$AB\bar{C} + B\bar{C} + \bar{B}A \quad \text{(absorption)}$$
$$= \underline{\underline{B\bar{C} + \bar{B}A}}$$

(c) Draw the simplified logic gate circuit.




## Assignment 2

Decide whether each of the following problems (a - d) involves a permutation or a combination and then work out the answer.

(a) How many 4 digit numbers can be made from the digits $2, 3, 5, 6, 7$ and $9$ if no repetition of digits is allowed.

This is a permutation of subsets problem. The number of ways to choose 4 digits from 6 is $P(6,4) = \frac{6!}{(6-4)!} = \underline{\underline{360}}$.

(b) A student has to answer 8 out of 10 questions in an exam. How many different choices has she?

This is a combination problem. The number of ways to choose 8 questions from 10 is $C(10,8) = \frac{10!}{8!(10-8)!} = \underline{\underline{45}}$.

(c) How many different car number plates can be made if each plate contains 2 different letters (A-Z) followed by 5 distinct digits (0-9)?

This is a permutation of subsets problem. The number of ways to choose 2 letters from 26 is $P(26,2) = \frac{26!}{(26-2)!} = 650$. The number of ways to choose 5 digits from 10 is $P(10,5) = \frac{10!}{(10-5)!} = 30240$. The total number of ways to choose 2 letters and 5 digits is $650 \times 30240 = \underline{\underline{19656000}}$.

(d) How many ways are there of playing a game of lotto requiring you to select 7 correct numbers out of 36?

This is a combination problem. The number of ways to choose 7 numbers from 36 is $C(36,7) = \frac{36!}{7!(36-7)!} = \underline{\underline{8347680}}$

(e) Assuming you only play one game of lotto as described in (d), what is the probability of winning?

The probability of winning is the number of ways to win divided by the total number of ways to play the game. The probability of winning is $\frac{1}{8347680} = \underline{\underline{0.00000012}}$. Good luck!

## Assignment 3

The following table summarises visits to emergency departments at four hospitals in Denmark. People may leave without being seen by a physician, and those visits are denoted as LWBS. The remaining visits are serviced at the emergency department, and the visitor may or may not be admitted for a stay in the hospital

| | Hospital | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **Total** |
| Total | 5292 | 6991 | 5640 | 4329 | 22,252 |
| LWBS | 195 | 270 | 246 | 242 | 953 |
| Admitted | 1277 | 1558 | 666 | 984 | 4485 |
| Not admitted | 3820 | 5163 | 4728 | 3103 | 16,814 |

(a) Let $A$ denote the event that a visit is to **hospital 1**, and let $B$ denote the event that the result of the visit is LWBS (at any hospital). Find the number of outcomes in $A \cap B$, $A^c$, and $A \cup B$.

$$|A \cap B| = \underline{\underline{195}}$$
$$|A^c| = 22252 - 5292 = \underline{\underline{16960}}$$
$$\begin{aligned} |A \cup B| &= |A| + |B| - |A \cap B| \\ &= 5292 + 953 - 195 \\ &= \underline{\underline{6050}} \end{aligned}$$

(b) Now, let $A$ denote the event that a visit is to **hospital 4**, and let $B$ denote the event that a visit results in LWBS (at any hospital). Determine the following probabilities. $P(A \cap B)$, $P(A^c)$, $P(A \cup B)$, $P(A \cup B^c)$, $P(A^c \cap B^c)$

$$P(A \cap B) = \frac{242}{22252} = \underline{\underline{0,0109}}$$
$$P\left(A^C\right) = \frac{22252 - 4329}{22252} = \underline{\underline{0.805}}$$
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{4329 + 953 - 242}{22252} \\ &= \underline{\underline{0.226}} \end{aligned}$$

## Assignment 4

Heart failures are due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances (73%) or foreign objects (27%). Natural occurrences are caused by arterial blockage (56%), disease (27%), and infection (e.g., staph infection) (17%).

(a) Determine the probability that a failure is due to an induced substance.

The probability that a failure is due to an induced substance is $0.13 \times 0.73 = \underline{\underline{0.0949}}$.

(b) Determine the probability that a failure is due to disease or infection.

The probability that a failure is due to disease or infection is $0.87(0.27 + 0.17) = \underline{\underline{0.3828}}$.

## Assignment 5

Let $A$ and $B$ be two events such that:

$$P(A) = 0.4, \quad P(B) = 0.7, \quad P(A \cup B) = 0.9$$

(a) Find $P(A \cap B)$.

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.4 + 0.7 - 0.9 \\ &= \underline{\underline{0.2}} \end{aligned}$$

(b) Find $P(A^c \cap B)$.

$$\begin{aligned} P(A^c \cap B) &= P(B - A) \\ &= P(B) - P(A \cap B) \\ &= 0.7 - 0.2 \\ &= \underline{\underline{0.5}} \end{aligned}$$

(c) Find $P(A - B)$.

$$\begin{aligned} P(A - B) &= P(A) - P(A \cap B) \\ &= 0.4 - 0.2 \\ &= 0.2 \end{aligned}$$

(d) Find $P(A^c - B)$.

$$\begin{aligned} P(A^c - B) &= P(A^c) - P(A^c \cap B) \\ &= 1 - P(A) - P(A^c \cap B) \\ &= 1 - 0.4 - 0.5 \\ &= \underline{\underline{0.1}} \end{aligned}$$

(e) Find $P(A^c \cup B)$.

$$\begin{aligned} P(A^c \cup B) &= P(A^c) + P(B) - P(A^c \cap B) \\ &= P(A^c) - P(A^c \cap B) + P(B) \\ &= P(A^c - B) + P(B) \\ &= 0.1 + 0.7 \\ &= \underline{\underline{0.8}} \end{aligned}$$

(f) Find $P(A \cap (B \cup A^c))$.

$$\begin{aligned} P(A \cap (B \cup A^c)) &= P(A) + P(A^c \cup B) - P(A \cup A^c \cup B) \\ &= 0.4 + 0.8 - 1 \\ &= \underline{\underline{0.2}} \end{aligned}$$

## Assignment 6

Four teams $A$, $B$, $C$, and $D$ compete in a tournament. Teams $A$ and $B$ have the same chance of winning the tournament. Team $C$ is twice as likely to win the tournament as team $D$. The probability that either team $A$ or team $C$ wins the tournament is 0.6. Only one team can win the tournament. Find the probabilities of each team winning the tournament.

$$P(A) = P(B)$$
$$P(C) = 2 \cdot P(D)$$
$$P(A \cup C) = 0.6$$

Since only one team can win:

$$P(A \cup C) = P(A) + P(C) = 0.6$$
$$P(B \cup D) = P(B) + P(O) = P(\overline{A \cup C}) = 1 - 0.6 = 0.4$$
$$P(D) = 0.4 - P(B) = 0.4 - P(A)$$
$$\begin{aligned} P(C) = 0.6 - P(A) &= 2 \cdot P(D) \\ 0.6 - P(A) &= 2 \cdot (0.4 - P(A)) \\ 0.6 - P(A) &= 0.8 - 2P(A) \\ P(A) &= 0.8 - 0.6 = 0.2 \end{aligned}$$

Therefore:

$$P(A) = P(B) = \underline{\underline{0.2}}$$
$$P(C) = 0.6 - P(A) = \underline{\underline{0.4}}$$
$$P(D) = \frac{1}{2} \cdot P(A) = \underline{\underline{0.2}}$$