MSE Hand in 3

The assignments are to be solved in pairs. Each pair can only hand in one solution. Submission by the end of class. The assignments must be solved by hand.

Assignment 1

These are recap exercises from last week's topic.

(a) What is $F(1337_8)$ where F is defined as $F(x) = x_{16}$?

(b) What is the result of $10_{16} + 100_{10} + 1000_8 + 10000_2$?

Solution

(a) First, 1337_8 to decimal:

$$\begin{aligned} 1337_8 &= 1 \times 8^3 + 3 \times 8^2 + 3 \times 8^1 + 7 \times 8^0 \\ &= 512 + 192 + 24 + 7 = 735_{10} \end{aligned}$$

Then, 735_{10} to hexadecimal:

$$\begin{aligned} 735 \div 16 &= 45 \text{ remainder } 15 \quad (F) \\ 45 \div 16 &= 2 \text{ remainder } 13 \quad (D) \\ 2 \div 16 &= 0 \text{ remainder } 2 \end{aligned}$$

Reading the remainders in reverse order:

$$2DF_{16}$$

Alternatively, 1337_8 to 3-bit binary:

$$\begin{aligned} 1_8 &= 001_2 \\ 3_8 &= 011_2 \\ 3_8 &= 011_2 \\ 7_8 &= 111_2 \end{aligned}$$

Gathering bits:

$$1337_8 = 001011011111_2$$

Then, for each 4-bit nibble, convert to hexadecimal:

$$\begin{aligned} 0010_2 &= 2_{16} \\ 1101_2 &= D_{16} \\ 1111_2 &= F_{16} \end{aligned}$$

Gathering hexadecimals:

$$2DF_{16}$$

Therefore, $F(1337_8) = 2DF_{16}$.

(b) Convert each number to decimal:

$$\begin{aligned} 10_{16} &= 1 \times 16^1 + 0 \times 16^0 = 16 \\ 100_{10} &= 100 \\ 1000_8 &= 1 \times 8^3 + 0 \times 8^2 + 0 \times 8^1 + 0 \times 8^0 = 512 \\ 10000_2 &= 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 16 \end{aligned}$$

Add the numbers:

$$16 + 100 + 512 + 16 = 644$$

Therefore, the result is 644_{10} (or 284_{16} or 1204_8 or 1010000100_2).

Assignment 2

Calculate

(a) $1 \cdot \overline{0}$

(b) $1 + \overline{1}$

(c) $\overline{1 + 0 \cdot 1}$

Solution

(a) $\overline{0} = 1$, so $1 \cdot 1 = 1$.

(b) $\overline{1} = 0$, so $1 + 0 = 1$.

(c) $0 \cdot 1 = 0$, so $1 + 0 = 1$, then $\overline{1} = 0$.

Assignment 3

Use truth tables to show all the possible inputs and outputs of

(a) $F(x) = x \cdot \overline{x} + (x + \overline{x})$

(b) $G(x, y) = \overline{x} \cdot \overline{y} + \overline{x \cdot y}$

(c) $H(x, y, z) = \overline{x} \cdot y + \overline{z}$

Solution

(a)

x	$\overline{x}$	$x \cdot \overline{x}$	$x + \overline{x}$	$F(x)$
0	1	0	1	$0 + 1 = 1$
1	0	0	1	$0 + 1 = 1$

Therefore, $F(x) = 1$ for all values of x.

(b)

x	y	$\overline{x}$	$\overline{y}$	$\overline{x} \cdot \overline{y}$	$x \cdot y$	$\overline{x \cdot y}$	$G(x, y)$
0	0	1	1	1	0	1	$1 + 1 = 1$
0	1	1	0	0	0	1	$0 + 1 = 1$
1	0	0	1	0	0	1	$0 + 1 = 1$
1	1	0	0	0	1	0	$0 + 0 = 0$

Therefore, $G(x, y) = 1$ unless both $x = 1$ and $y = 1$, in which case $G(x, y) = 0$.

(c)

x	y	z	$\overline{x}$	$\overline{z}$	$\overline{x} \cdot y$	$H(x, y, z)$
0	0	0	1	1	0	$0 + 1 = 1$
0	0	1	1	0	0	$0 + 0 = 0$
0	1	0	1	1	1	$1 + 1 = 1$
0	1	1	1	0	1	$1 + 0 = 1$
1	0	0	0	1	0	$0 + 1 = 1$
1	0	1	0	0	0	$0 + 0 = 0$
1	1	0	0	1	0	$0 + 1 = 1$
1	1	1	0	0	0	$0 + 0 = 0$

Therefore, $H(x, y, z) = 1$ when either $\overline{x} \cdot y = 1$ or $\overline{z} = 1$.

Assignment 4

Show that

(a) $x \cdot y + \overline{x} \cdot y = y$

(b) $x + y \cdot (\overline{x} + \overline{y}) = x + y$

(c) $x \cdot y \cdot z + \overline{x \cdot y \cdot z} = 1$

Solution

(a)

$$\begin{aligned} x \cdot y + \overline{x} \cdot y &= y \cdot (x + \overline{x}) && \text{(factor out } y\text{)} \\ &= y \cdot 1 && \text{(complement law: } x + \overline{x} = 1\text{)} \\ &= y && \text{(identity property of multiplication)} \end{aligned}$$

(b)

$$\begin{aligned} x + y \cdot (\overline{x} + \overline{y}) &= x + y \cdot \overline{x} + (y \cdot \overline{y}) && \text{(distribute } y\text{)} \\ &= x + y \cdot \overline{x} + 0 && \text{(complement law: } y \cdot \overline{y} = 0\text{)} \\ &= x + y \cdot \overline{x} && \text{(identity property of addition)} \\ &= (x + y) \cdot (x + \overline{x}) && \text{(distributive property)} \\ &= (x + y) \cdot 1 && \text{(complement law: } x + \overline{x} = 1\text{)} \\ &= x + y && \text{(identity property of multiplication)} \end{aligned}$$

(c)

$$\begin{aligned} x \cdot y \cdot z + \overline{x \cdot y \cdot z} &= (x \cdot y \cdot z) + \overline{(x \cdot y \cdot z)} && \text{(rewrite for clarity)} \\ &= 1 && \text{(complement law: } a + \overline{a} = 1\text{)} \end{aligned}$$

Assignment 5

An airline denies boarding for passengers if they 1) have no ticket, unless they paid extra or arrived early, 2) did not arrive early, unless they paid extra or are not a child, 3) are a child, unless they paid extra or have a ticket, 4) said "bomb", unless they paid extra or are a child.

(a) Define the non-negated boolean variables needed (e.g. "let T be 1 if the passenger has a ticket, 0 if not") to represent the above criteria for boarding.

(b) Define a single boolean function F that takes the variables from (a) and returns true if the passenger is allowed boarding, considering all four (1-4) stated criteria for denial.

(c) Simplify the function F from (b), and identify in which cases passengers are allowed to board.

Solution

(a) Define the variables:

- T: 1 if the passenger has a ticket, 0 otherwise.
- E: 1 if the passenger arrived early, 0 otherwise.
- P: 1 if the passenger paid extra, 0 otherwise.
- C: 1 if the passenger is a child, 0 otherwise.
- B: 1 if the passenger said "bomb", 0 otherwise.

(b) Based on the criteria for denial, express the necessary conditions for boarding as:

$$
\begin{aligned}
A_1 &= T + P + E && \text{(has ticket, or paid extra, or arrived early)} \\
A_2 &= E + P + \overline{C} && \text{(arrived early, or paid extra, or is not a child)} \\
A_3 &= \overline{C} + P + T && \text{(is not a child, or paid extra, or has ticket)} \\
A_4 &= \overline{B} + P + C && \text{(did not say "bomb", or paid extra, or is a child)}
\end{aligned}
$$

All necessary conditions for boarding must be met. Therefore, the overall function is:

$$F = A_1 \cdot A_2 \cdot A_3 \cdot A_4 = (T + P + E)(E + P + \overline{C})(\overline{C} + P + T)(\overline{B} + P + C)$$

(c) Simplifying the function F:

$$
\begin{aligned}
F &= (T + P + E)(E + P + \overline{C})(\overline{C} + P + T)(\overline{B} + P + C) \\
&= P + (T + E)(E + \overline{C})(\overline{C} + T)(\overline{B} + C) && \text{(factor out } P\text{)} \\
&= P + (TE + T\overline{C} + E + E\overline{C})(\overline{C} + T)(\overline{B} + C) && \text{(distribute } (T + E) \text{ over } (E + \overline{C})\text{)} \\
&= P + (TE + TE\overline{C} + T\overline{C} + E\overline{C})(\overline{B} + C) && \text{(distribute } (\overline{C} + T)\text{)} \\
&= P + TE\overline{B} + TE\overline{CB} + T\overline{CB} + E\overline{CB} + TEC && \text{(distribute } (\overline{B} + C)\text{, } C \text{ negates } \overline{C}\text{)} \\
&= P + TE\overline{B} + \overline{CB}(TE + TE + T + E) + TEC && \text{(factor out } \overline{CB}\text{)} \\
&= P + \overline{CB}(TE + TE + T + E) + TEC && (\overline{CB}(TE) + TEC \text{ covers } TE\overline{B}) \\
&= P + \overline{CB}(T + E) + TEC && \text{(absorb } TE \text{ and simplify)}
\end{aligned}
$$

The final simplified function is:

$$F = P + \overline{CB}(T + E) + TEC$$

We conclude that passengers will be allowed boarding, if either of the following conditions are met:

- They paid extra.
- They have a ticket, or are early, given that they are not children, and did not say "bomb".
- They have a ticket, and are early, and are children.