MSE Hand in 2 - solution

The assignments are to be solved in pairs. Each pair can only hand in one solution. Submission by the end of class. The assignments must be solved by hand.

Assignment 1

These are recap exercises from last week's topic.

(a) Find $\gcd(75, 240)$.

Solution:

To find the greatest common divisor (gcd) of 75 and 240, we'll use the **prime factorization method**.

Step 1: Prime Factorization of Each Number

Prime factors of 75:

$$75 = 3 \times 5 \times 5 = 3^1 \times 5^2$$

Prime factors of 240:

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 2^4 \times 3^1 \times 5^1$$

Step 2: Identify Common Primes and Their Minimum Exponents

Common primes: 3 and 5

Minimum exponents: For 3: $\min(1, 1) = 1$

For 5: $\min(2, 1) = 1$

Step 3: Compute the GCD

$$\gcd(75, 240) = 3^1 \times 5^1 = 3 \times 5 = 15$$

Final Answer:

$$\gcd(75, 240) = 15$$

(b) What is $\varphi(187)$?

Solution:

The function $\varphi(n)$ denotes Euler's totient function, which counts the positive integers up to n that are relatively prime to n.

Step 1: Prime Factorisation of 187

First, factorise 187:

$$187 = 11 \times 17$$

Step 2: Apply Euler's Totient Function Formula For a number n with prime factors p and q:

$$\varphi(n) = (p - 1)(q - 1)$$

Step 3: Compute $\varphi(187)$

Substitute $p = 11$ and $q = 17$:

$$\varphi(n) = (11 - 1)(17 - 1) = 160$$

Assignment 2

(a) Convert the decimal number 57 to binary.

Solution: We perform successive divisions by 2 and record the remainders:

$$\begin{aligned}
57 \div 2 &= 28 \text{ with remainder } 1 \\
28 \div 2 &= 14 \text{ with remainder } 0 \\
14 \div 2 &= 7 \text{ with remainder } 0 \\
7 \div 2 &= 3 \text{ with remainder } 1 \\
3 \div 2 &= 1 \text{ with remainder } 1 \\
1 \div 2 &= 0 \text{ with remainder } 1
\end{aligned}$$

Reading the remainders from bottom to top, we get:

$$57_{10} = 1\,1\,1\,0\,0\,1_2 = 111001_2$$

Final Answer:

$$57_{10} = 111001_2$$

(b) Convert the decimal number 57 to Hexadecimal. You may depart from your answer to question (a).

Solution:

From question (a), we have:

$$57_{10} = 111001_2$$

To convert binary 111001_2 to hexadecimal, group the binary digits into groups of four bits from right to left:

$$111001 \rightarrow 0011\,1001$$

Convert each group to hexadecimal:

- $0011_2 = 3_{16}$

- $1001_2 = 9_{16}$

Thus:

$$57_{10} = 39_{16}$$

Final Answer:

$$57_{10} = 39_{16}$$

(c) Calculate $11010_2 + 1101_2$ in binary

$$\begin{array}{rrrrrrr}
 & & {\scriptstyle 1} & & & & \\
 & & 1 & 1 & 0 & 1 & 0 \\
+ & & & 1 & 1 & 0 & 1 \\
\hline
 & 1 & 0 & 0 & 1 & 1 & 1
\end{array}$$

(d) Calculate $100_2 \cdot 111_2$ in binary.

$$\begin{array}{rrrrrr}
 & & & 1 & 0 & 0 \\
\times & & & 1 & 1 & 1 \\
\hline
 & & & 1 & 0 & 0 \\
 & & 1 & 0 & 0 & \\
+ & 1 & 0 & 0 & & \\
\hline
 & 1 & 1 & 1 & 0 & 0
\end{array}$$

Assignment 3

(a) Find the binary and the hexadecimal **expansion** of the decimal number 1337.

Solution:

$$1337 = 2^{10} + 2^8 + 2^5 + 2^4 + 2^3 + 2^0 = 10100111001_2$$

Hexadecimal Expansion:

$$1337 = 5 \times 16^2 + 3 \times 16^1 + 9 \times 16^0 = 539_{16}$$

Final Answer:

$$1337_{10} = 10100111001_2 = 539_{16}$$

(b) Find the binary and the decimal representation of the hexadecimal number $F1D0_{16}$.

Solution:

Binary Expansion:

Convert each hexadecimal digit to binary:

$$F = 1111, \quad 1 = 0001, \quad D = 1101, \quad 0 = 0000$$

Thus:

$$F1D0_{16} = 1111\ 0001\ 1101\ 0000_2$$

Decimal Expansion: Convert each digit to its decimal equivalent and expand:

$$F1D0_{16} = 15 \times 16^3 + 1 \times 16^2 + 13 \times 16^1 + 0 \times 16^0$$

Calculating each term:

$$15 \times 4096 = 61{,}440, \quad 1 \times 256 = 256, \quad 13 \times 16 = 208, \quad 0 \times 1 = 0$$

Sum:

$$61{,}440 + 256 + 208 = 61{,}904$$

Final Answer:

$$F1D0_{16} = 1111000111010000_2 = 61{,}904_{10}$$

(c) Multiply the numbers 10010_2 and 100101_2 in binary.

Solution:

```
        10010
     × 100101
     --------
        10010
       00000
      10010
     00000
    00000
 + 10010
 ------------
   1010011010
```

(d) Convert your answer from exercise (c) to hexadecimal and decimal.

Solution:

$1010 = 10_{10} = A_{16}$
$1001 = 9_{10} = 9_{16}$
$0010 = 2_{10} = 2_{16}$
$1010011010 = 29A_{16}$

$$1010011010_2 = 1 \cdot 2^9 + 0 \cdot 2^8 + 1 \cdot 2^7 + 0 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 666_{10}$$

Assignment 4

Let H be the set of all hexadecimal numbers with 2 digits, let B be the set of all binary numbers with 9 digits, and let f be a function from H to B such that $f(x) = x_2$.

(a) What is the order of H ?

Solution:

- Each digit in a hexadecimal number can take any value from 0 to F, which corresponds to the decimal values 0 to 15 .
- Since there are two digits, the total number of elements in H is:

Order of $H = 16 \times 16 = 256$

(b) What is the order of B ?

Solution:

- Each digit in a binary number can be either 0 or 1 .
- For 9-digit binary numbers, the total number of combinations is:

Order of $B = 2^9 = 512$

(c) Determine $f(A3)$.

Solution:

$3_{16} = 0011$ and $A_{16} = 1010$, so $f(A3) = 10100011_2$

Assignment 5

A calculator has space for 4 digits on its screen. In the exercises below, assume you have all the characters from 0-9 and from A to F available

(a) What is the largest decimal number you can write on the calculator? 9999

(b) What is the largest binary number you can write on the calculator? State your result both in binary and in decimal. 1111_2 and 15_{10}

(c) What is the largest hexadecimal number you can write on the calculator? State your result both in hexadecimal and in decimal, and then try to figure out (e.g. by googling!) why this number is important in the field of computing. $FFFF$

(d) State the number 65537_{10} in binary and then try to figure out why this number is so very important in public key cryptography (e.g. by googling!). 10000000000000001_2